Spectral content of NRZ test patterns

Non-return-to-zero (NRZ) signaling is widely used for data transmission in digital communication systems. Many NRZ test patterns have been created for system test and verification. These patterns are usually designed either to simulate actual data or to stress certain aspects of the system. To understand the effects of the various test patterns on a particular system, it is important to understand the frequency characteristics of both the test pattern and the system under test.

This article shows straightforward relationships between the time-domain characteristics of NRZ test patterns, such as data rate and pattern length, and their frequency-domain spectral components. Topics include an overview of NRZ test patterns, computation of the power spectrum, lab measurements of the power spectrum, and application of these concepts to system understanding.

Overview of NRZ test patterns

In NRZ signaling, each binary bit is assigned a unique time slot of duration, called the bit period (T_b). The signal is either high (representing a one) or low (representing a zero) during the entire bit period. NRZ waveforms are defined and measured as functions of time. Specifically, they are time-domain signals.

For a random NRZ data stream, each bit in the sequence has an equal probability (50%) of being a one or a zero, regardless of the state of the preceding bit(s). It is therefore possible to have large sequences of consecutive identical digits (CIDs). Because of the very low frequency content produced by long sequences of CIDs in the data signal, designing high-speed systems that can work with random data can be difficult.

Data encoding, or scrambling, is often used to format the random data into a more manageable form. One of the most widely used encoding methods in high-speed systems is known as 8b10b, which is used in Ethernet, Fibre Channel, and high-speed video applications. 8b10b encoding takes 8 bits of data and replaces it with a 10-bit symbol. The extra bits are added to balance the pattern (make the number of ones equal the number of zeros for a given bit interval) and limit the maximum number of CIDs. The encoding algorithm is also used to improve bit-error ratio (BER) by mapping the 8 bits to specific symbols in the 10-bit signal space that can be easily distinguished from other 10-bit symbols. Other methods, such as scrambling or 64b66b encoding, are common to SONET and SDH telecommunication systems. Scrambling and 64b66b encoding also work to balance the pattern and improve the BER, though much larger runs of CIDs are possible with these methods.

For a given application, there may be several types of test patterns that stress various performance aspects or system components. For example, a K28.5± pattern (11000001010011111010) is often used to test the deterministic jitter performance of systems that use 8b10b encoding. Likewise, a pseudorandom bit stream (PRBS) is used as a general-purpose test pattern in encoded, random, and scrambled NRZ applications.

The PRBS is typically denoted as a $2^X - 1$ PRBS. The power (X) indicates the shift register length used to create the pattern. Each $2^X - 1$ PRBS contains every possible combination of X number of bits (except one). A short PRBS, such as the $2^7 - 1$ PRBS (127 bits), is often used in Ethernet, Fibre Channel, and high-speed video applications, because it provides a good approximation to an 8b10b-encoded NRZ data stream. A $2^{23} - 1$ (≈ 8.4 million bits) PRBS is commonly used in both SONET and SDH telecommunication systems, which require a test pattern with lower frequency content and provide a better representation of scrambled or random NRZ data.

Computing the power spectrum of an NRZ test pattern

Each NRZ test pattern has an associated power spectral density (PSD) that indicates the frequency distribution of the power in the pattern. The two primary methods of computing PSD are to: (a) square the magnitude of the Fourier transform of the pattern; or (b) compute the Fourier transform of the pattern's autocorrelation function[1]. The first method, (a), is generally simpler for signals that can be mathematically written in a finite, closed form (e.g., $s[t] = A\cos[2\pi f_0 t]$). The second method, (b), is used for more complicated signals, such as long sequences of NRZ data (like test patterns) or random bit streams. To apply these methods, a review of some of the basics of Fourier analysis[2] is useful.

- The delta function, $A\delta(t)$, can be thought of as an infinitely narrow rectangular pulse with area (A). It has a nonzero value only when the argument of the function is equal to zero and is represented graphically by a vertical arrow.
- The comb function, $A\Sigma_n\delta(t - nT)$, is composed of an infinite number of equal-area delta functions spaced at uniform intervals (T).

- The Fourier transform of a comb function is also a comb function, where the interval is inverted (e.g., n/T) and the areas of the delta functions are modified by the inverted interval (e.g., A/T).
- Convolution in the time domain (represented symbolically by *) is equivalent to multiplication in the frequency domain, and vice versa.
- Convolution of a signal with a delta function results in a copy of the signal that is shifted to the location of the delta function.
- Multiplication of a signal with a delta function, or "sampling", results in a delta function with an area modified by the signal's magnitude at the location of the delta function.

As an example of an application of the above rules, the PSD of an NRZ test pattern is computed (**Figure 1**). The test pattern can be represented by a sequence of high and low levels (representing ones and zeros) with a defined T_b and total pattern length, $L = nT_b$. An infinite repetition of the pattern results from the convolution of the finite-length test pattern with a comb function that has a spacing interval equal to the pattern length (Figure 1a). Next, the autocorrelation functions for each component of the test pattern are separately computed (Figure 1b). Note that the autocorrelation of the test pattern approximates a triangle (the accuracy of this approximation improves as the length and "randomness" of the pattern increase). Finally, the Fourier transform of the autocorrelation functions are used to compute the power spectrum (Figure 1c).

The power spectrum resulting from the example in Figure 1 shows an infinite sequence of discrete spectral lines (delta functions) scaled by a "$\text{sinc}^2(f)$" envelope, where sinc(f) is defined as $\sin(\pi f)/(\pi f)$. Important observations that apply to test patterns in general include: (a) the nulls in the $\text{sinc}^2(f)$ envelope occur at integer multiples of the data rate; (b) spectral lines are evenly spaced at an interval that is the inverse of the pattern length; and (c) the magnitude of the $\text{sinc}^2(f)$ envelope decreases (i.e., "flattens out") as the data rate and/or pattern length increase. In the limit, as the pattern length approaches infinity, the spacing between the spectral lines becomes infinitesimally small, and the spectrum shape approaches a continuous $\text{sinc}^2(f)$ function.

As an example, if the 6-bit pattern shown in Figure 1a is transmitted at a data rate of 1.25Gbps, the spectral-line spacing, amplitude, and spectral nulls can then be calculated as shown in **Figure 2**. Note that $\text{sinc}^2(f)$ envelope shown in Figure 2 is an approximation of the 6-bit pattern. The accuracy of this approximation improves as the pattern length or randomness increases.

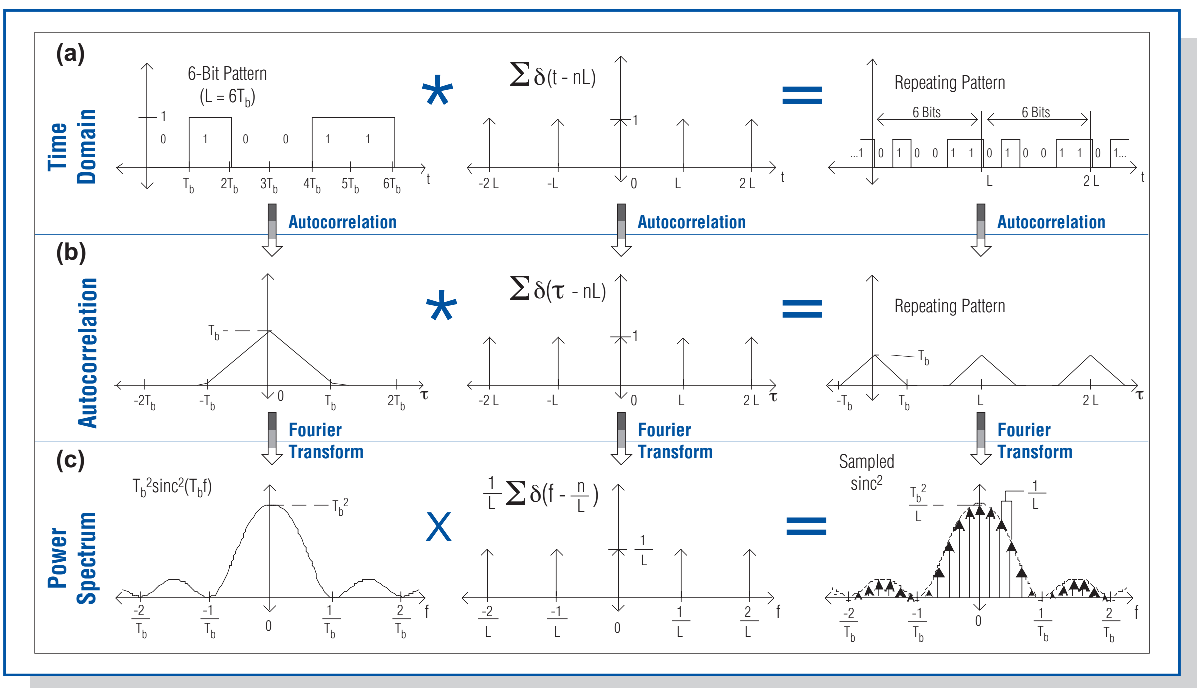

Figure 1. The test patterns illustrate time domain (a), autocorrelation (b), and power spectrum (c) of NRZ signaling.

Spectrum analyzer measurements

The equations and principles described previously can also be demonstrated by lab measurement. This involves using a high-speed pattern generator to create the test patterns, and a spectrum analyzer to measure the PDS of the signal. Starting with a simple example, the measured spectrum of a 4-bit pattern (1110 test pattern) transmitted at 1.25Gbps can be seen in **Figure 3**. The spectral nulls are measured at 1.25GHz ($1/T_b$) and 2.5GHz ($2/T_b$), and the line spacing is 312.5GHz (1/L). The power spectrum envelope is also seen to be approximately $\text{sinc}^2(f)$. The slight deviations in magnitude result from the short pattern used in this example.

Increasing the pattern length to 20 bits (K28.5± test pattern) and keeping the transmission rate at 1.25Gbps (**Figure 4**), the spectral nulls are measured to be in the same locations (1.25GHz and 2.5GHz). Meanwhile, the spectral-line spacing is reduced to 125MHz due to the longer pattern length. The $\text{sinc}^2(f)$ envelope of the spectral lines is also a more accurate representation. Additionally, the envelope of the spectral lines more closely matches the $\text{sinc}^2(f)$ function than the 4-bit pattern example.

The K28.5± test pattern presents an interesting aspect to this topic. It is a 20-bit pattern. However, the spectral-line spacing is measured to be 125MHz when transmitted at 1.25Gbps, which would correspond to a 10-bit test pattern. This discrepancy is because the K28.5± pattern is composed of a K28.5+ sequence (1100000101) and its inverse, the K28.5- sequence (0011111010). In the frequency domain, the K28.5- sequence contains the same spectral information as the K28.5+ sequence. The pattern therefore repeats spectrally every 10 bits, which is the reason for the 125MHz spacing in Figure 4.

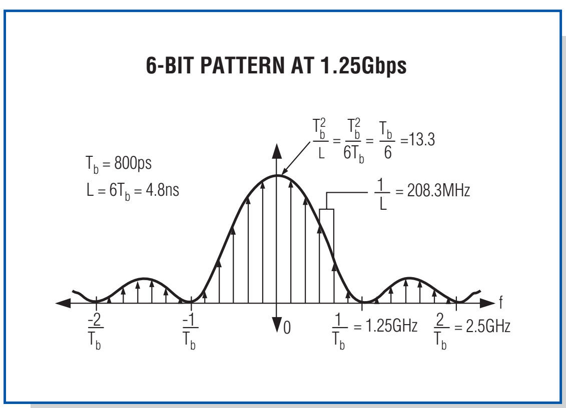

Figure 2. The approximate power spectrum of a 6-bit NRZ pattern shows spectral-line spacing and the $\text{sinc}^2(f)$ envelope.

The $\text{sinc}^2(f)$ envelope becomes very apparent as the pattern length is increased further. **Figure 5** illustrates this point by using a 2^7 - 1 PRBS pattern (127 bits) transmitted at 2.5Gbps. At this longer pattern length, delta spacing is reduced to approximately 19.7MHz. And, corresponding to the higher data rate, spectral nulls are at 2.5GHz and 5GHz. Given the small spectral-line spacing in respect to the data rate, the $\text{sinc}^2(f)$ envelope and spectral nulls are clearly seen in the power spectrum (Figure 5).

Figure 6 illustrates the difference in the spectral-line magnitude and spacing for a 2^7 - 1 PRBS pattern at 1.25Gbps and 2.5Gbps. As seen in this figure, when

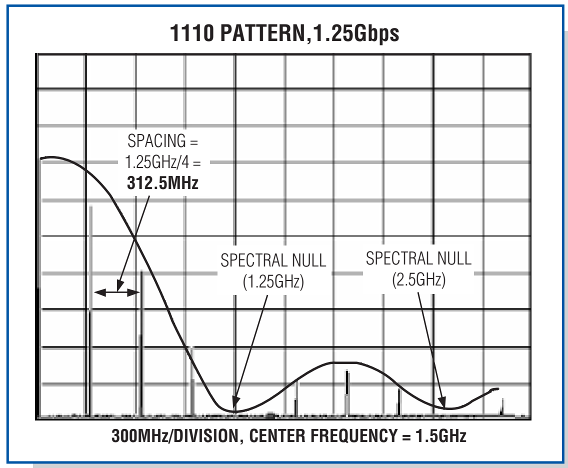

Figure 3. The power spectrum of a 4-bit pattern shows slight deviations of the spectral-line magnitude from the sinc2(f) envelope. As the pattern length increases, the deviation is reduced.

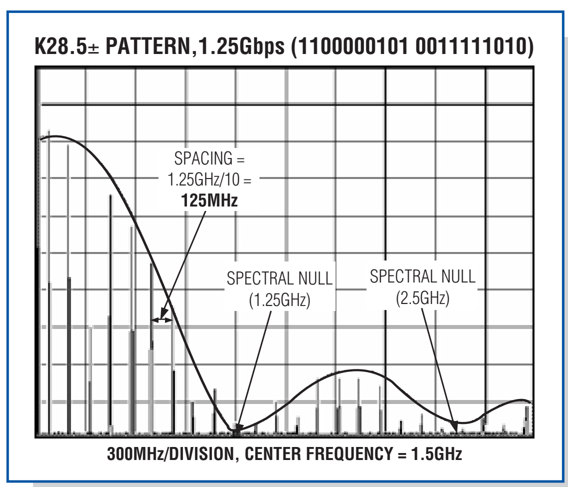

Figure 4. The measured power spectrum of a K28.5± test pattern shows the improved approximation of the sinc2(f) envelope and the reduced spectral-line spacing due to the longer pattern.

measured at the same frequency, the magnitude of the spectral lines and spacing is larger at 2.5Gbps data transmission rates than at 1.25Gbps.

Application examples

Knowledge of the power spectrum of NRZ test patterns can lead to significant improvements in digital communication-system design. This is illustrated through examples of three different applications: receiver bandwidth, adaptive equalizers, and electromagnetic interference (EMI).

Receiver bandwidth

The design process for a receiver inevitably includes questions about the necessary bandwidth. If the bandwidth is too low, the high-frequency components of the received signal are attenuated, and the signal is distorted. If the bandwidth is too high, excess noise is admitted to the receiver, causing a reduction in signal-to-noise ratio (SNR). Also, to achieve the higher bandwidth[3], an increase in complexity and cost is necessary. Knowing the spectral content of the signals that will be received, the bandwidth decision can be made in a manner that includes only the critical spectral components.

Adaptive equalizer

Adaptive equalizers are designed to reverse distortion effects caused by nonideal transmission media. The MAX3800 adaptive cable equalizer, for example, reverses the distortion caused by skin-effect losses in copper cables at data rates as high as 3.2Gbps[4]. It accomplishes this by comparing the power of the input signal at two discrete frequencies (f_1 = 200MHz and f_2 = 600MHz).

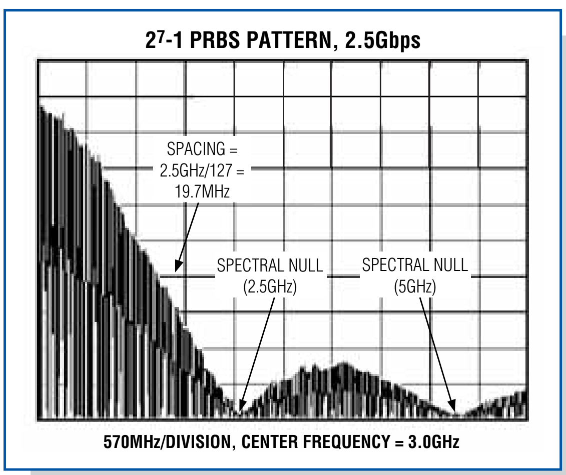

Figure 5. The power spectrum of the 2^7 - 1 PRBS (127 bits) clearly shows the spectral nulls and sinc2(f) envelope.

Based on the $sinc^2(T_b f)$ envelope of the power spectrum with the first null at 3.2GHz, the power ratio at these two frequencies should be $sinc^2(T_b f_1)$ / $sinc2(T_b f_2)$ = 0.987/0.890 = 1.11. If the measured ratio is different than expected, the equalizer changes the amount of skin-effect compensation in order to restore the correct ratio. This works well for high data rates and long data patterns. However, using our knowledge of spectral content of NRZ test patterns, we can predict that some patterns may cause problems.

If, for example, the data rate is reduced to 622Mbps, the $sinc^2(f)$ envelope with first null at 622MHz results in a 200MHz to 600MHz power detector ratio of 0.703/0.00134 = 525, instead of the expected 1.11. As the equalizer tries to restore the expected 1.11 power ratio, the output may be distorted. As another example, consider a short test pattern with a pattern length of 10 bits. For shorter patterns, the spectral lines are spaced at larger intervals. In the specific case of the 10-bit pattern at a data rate of 3.2Gbps, the spectral lines are separated by 320MHz, with the first few at 0, 320, and 640MHz. For this type of pattern and data rate, there may be little or no power to detect at 200MHz or 600MHz. This, in turn, can cause signal distortion, because the equalizer is not able to adapt correctly.

Electromagnetic interference (EMI)

The effects of EMI in a system can be reduced or eliminated by altering the magnitude and/or frequencies of the power spectrum. The alteration(s) can be done by changing the data rate or pattern length.

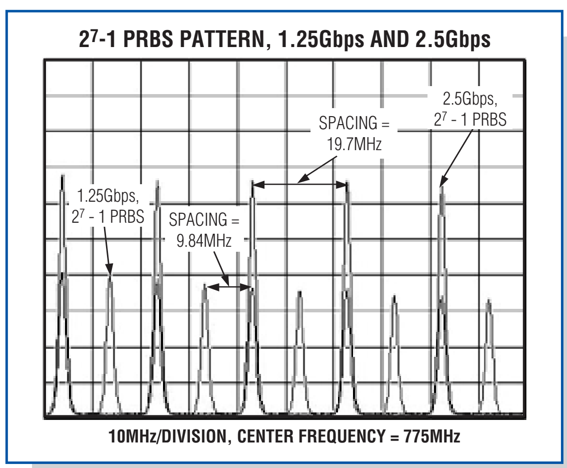

Figure 6. The measured power spectrum of a 2^7 - 1 PRBS pattern transmitted at 1.25Gbps and 2.5Gbps (as viewed at 725MHz to 825MHz) shows the spectral-line magnitude and spacing difference as the data rate changes.

As the data rate increases, the spectrum nulls are spread farther apart. Also, the magnitude of each spectral line is reduced by pushing some power to higher frequencies. Spreading the power over a larger frequency range leaves less at the frequencies of interest. One way to achieve this effect is by adding extra bits to the original data stream to effectively increase the data rate.

Pattern length also plays a role in EMI, because spectral-line magnitude and spacing vary as the pattern length changes. A longer pattern reduces the magnitude and spacing, while a shorter pattern increases the magnitude and spacing. To reduce EMI at a specific frequency, the pattern length can be changed to shift the spectral line away from a particularly sensitive frequency range. Alternatively, a longer pattern can be used to reduce the magnitude of the EMI.

Conclusion

A clear understanding of the frequency-domain spectral content of NRZ data is critical to success in high-speed, digital-communication system design. The principles presented in this article establish basic relationships between NRZ data-time domain characteristics (pattern length, data rate, etc.) and their corresponding frequency domain characteristics (spectral magnitude, envelope, and line spacing). These principles can be applied to a variety of circuit design issues, including filtering, signal equalization, and EMI.

References

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